# Mathematically Rich Games

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Here are some mathematically rich games you can play in your math classroom for your students to develop the skills of problem solving and investigation.

1. Tower of Hanoi
   The objective of the game is to move all the discs from one pole to another (there are a total of three poles) using the smallest number of moves possible. You can only move one disc at a time, and you cannot place a disc on top of a smaller one.
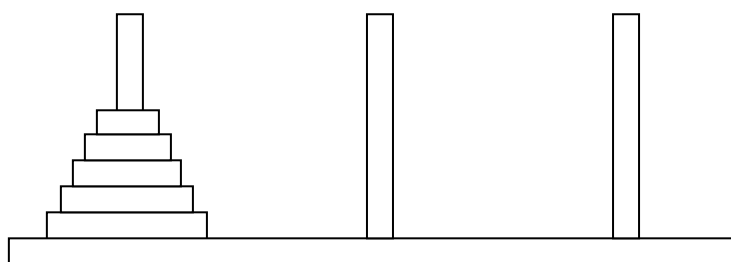
Heuristics:	Simplify the problem (start with a smaller number of discs)
	Solve part of the problem
	Working backwards
	Looking for patterns
	Logical reasoning
Thinking processes:	Specializing
	Conjecturing
	Justifying
	Generalizing
Extension:	What happens if there are four or more poles?

   There is actually a mathematical formula for the minimum number of moves. You can play this game online at www.mazeworks.com/hanoi

2. Nim
   There are many versions of this game. One version involves two players and a pile of 12 discs. Each player takes turn to pick either 1, 2 or 3 discs from the pile. The player who picks the last disc wins. This version has a sure-win strategy for the second player.

Heuristics:	Working backwards
	Looking for patterns
	Logical reasoning
Thinking processes:	Specializing
	Conjecturing
	Justifying
	Generalizing
Extension:	What happens if each player can pick 1, 2, 3 or 4 discs?
	What happens if there are a total of n discs and each player can

pick 1, 2, 3, … or *m* discs?

What happens if there are *p* piles of discs where each pile contains *n* discs each and each player can pick *any* number of discs from any one pile at any one time?

What happens if the *p* piles do not contain equal numbers of discs in each pile?

What happens if a pile of discs can be separated into two piles, depending on how a player picks a disc?

This last scenario is actually what happens in the following version of *Nim*, which is commonly played in schools. Each player can cancel any number of sticks in any one row at any one time. If the player cancels two sticks as shown below, the last row has effectively split into two rows, and a player cannot cancel "across the cancelled sticks".

This version also has a sure-win strategy for either the first or the second player, depending on how many starting rows of sticks there are. If there are 7 rows, as shown below, the second player will win if he knows the sure-win strategy.

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